MATH 611

Homework 8

1. Consider the case of generating observations from the sampling distribution of the median M, from a random sample of size n from the exponential density. Assuming that the sample size is odd verify that the sampling density of M is:
2. Write an R-function that computes the logarithm of the density in question 1.
3. Consider the proposal density

and write a function to implement the Metropolis-Hastings algorithm. Use an initial value of the MC equal to 1. The function should return a list of two components; the vector of simulated draws (for 10000 iterations) in the MC and the acceptance rate. Give the summary statistics of the simulated sample and the acceptance rate. Also, plot the densities of the exact and the simulated samples together.

1. Ron, Mary and Ted arrive at the beginning of their professor’s office hours. The amount of time they will stay is exponentially distributed with means 1, ½ and 1/3 hour. What is the probability that the first one is gone after 25 minutes?
2. The probability of a three of a kind in poker is approximately 1/50. Use the Poisson approximation to the binomial to estimate the probability you will get at least one three of a kind if you play 20 hands of poker.
3. A submarine has three navigational devices but can remain at the sea if at least two of them are working. Suppose that the failure times are exponential with means 1, 1.5 and 3 years. What is the average length of time the boat can remain at the sea?